P.6 Complex Numbers

Imaginary Number: $i = \sqrt{-1}$ and $i^2 = -1$

Complex Numbers: The set of all numbers of the form $a + bi$ where a and b are real numbers.

a is called the **real part** and b is called the **imaginary part**. If $b \neq 0$, then $a + bi$ is an **imaginary number.** The form $a + bi$ is called the **standard form** of a complex number.

Examples: Determine whether each complex number is real or imaginary and write it in standard form.

a) $4i$ b) $3 - 6i$ c) 5 d) $\dfrac{i - 3\pi}{4}$

Addition, Subtraction, and Multiplication of Complex Numbers.

$(a + bi) + (c + di) = (a + c) + (b + d)i$

$(a + bi) - (c + di) = (a - c) + (b - d)i$

$(a + bi)(c + di) = (ac - bd) + (bc + ad)i$ or use FOIL.

Examples: Perform the indicated operations.

a) $(6 + 2i) + (4 - 3i) =$ b) $(7 - 4i) - (-2 + 8i) =$

c) $(6 + 5i)(8 + 3i) =$ d) $(1 - i)(4 + i) =$

Powers of *i*:

Since $i^2 = -1$, $i^3 = i^2 \cdot i = -1 \cdot i = -i$, and $i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$

The first eight powers are listed here:

$i^1 = i$ $\quad$ $i^5 = i$

$i^2 = -1$ $\quad$ $i^6 = -1$

$i^3 = -i$ $\quad$ $i^7 = -i$

$i^4 = 1$ $\quad$ $i^8 = 1$

The powers of *i* continue in this pattern.

Examples: Simplify the power of i.

a) $i^{35} =$ b) $i^{29} =$ c) $i^{98} =$ d) $i^{48} =$

Theorem: If a and b are real numbers, then the product of $a+bi$ and its conjugate $a-bi$ is the real number a^2+b^2. $(a+bi)(a-bi)=(a^2+b^2)$

Examples: Find the product of the complex number and its conjugate.

a) $3-7i$ b) $2+9i$ c) i

Examples: Write each quotient in the form $a+bi$.

a) $\dfrac{6-2i}{3} =$ b) $\dfrac{2}{8+9i} =$ c) $\dfrac{4-5i}{3+2i} =$

Roots of Negative Numbers

For any positive real number b, $\sqrt{-b} = i\sqrt{b}$.

Examples: Write each expression in the form $a+bi$, where a and b are real numbers.

a) $\sqrt{-5}+\sqrt{-8} =$ b) $\sqrt{-20}\left(\sqrt{-6}-\sqrt{-4}\right) =$

c) $\dfrac{-2+\sqrt{-48}}{2} =$

Example: Does the complex number $x=1+3i\sqrt{2}$ satisfies the equation $x^2-2x+4=0$?